An Innovative Approach to Optimize Inbound Delivery Scheduling at Distribution Centers:
Application at a Large U.S. Restaurant Chain

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ABSTRACT

Distribution centers (DCs) run more efficiently if their inbound deliveries are spread out evenly over the days of the week so that volume is aligned with receiving capacity. However, delivery dates are often scheduled at the time of placing purchase orders without an efficient process for spreading deliveries evenly, and there has been little research on how to optimally do this. This paper terms this process Master Purchase Receipt Scheduling (MPRS) and introduces a novel MPRS algorithm developed by New Horizon Soft, LLC using data from a major U.S. restaurant chain, where the algorithm is now in use. The approach optimizes order delivery dates, accounting for vendor shipping calendars, shipment sizes, and transportation uncertainty. Results from a Monte Carlo simulation validate the effectiveness and robustness of the algorithm. By addressing the complexities of DC operations, this innovation promises to reduce receiving bottlenecks, increase efficiency and capacity utilization, and foster additional research in this vital area.

KEYWORDS

Inbound Delivery Scheduling, Inbound Freight Management, Inbound Logistics, Master Purchasing Receipt Schedule, Purchase Order Optimization, Receiving Scheduling, Transportation Scheduling

Optimal inbound delivery scheduling at distribution centers (DCs) is critical for their efficient operation. To make the best use of available loading docks, receiving personnel, and equipment, inbound deliveries should occur in a smooth, level pattern across the days of the week. To illustrate this point with an extreme example, it is preferable to receive 30 trucks each day rather than 210 trucks on Monday and no trucks for the rest of the week.

Most academic and industry research on DC scheduling has focused on outbound shipments because they have a more visible impact on the downstream customer. However, optimizing inbound shipment schedules is just as important to ensure that deliveries are readily available for customer orders. Ineffective scheduling can lead to a cascade of logistical inefficiencies, including DC congestion, high labor costs, contractual detention fees, delayed outbound shipments, and customer dissatisfaction.

Inbound shipment scheduling typically requires two stages:

1. High Level: Master Purchasing Receipt Scheduling (MPRS). We've introduced the term MPRS (where the "S" stands for schedule or scheduling, depending on context) to refer to the scheduling of purchase order (PO) due dates established when a buyer places POs. This is conceptually similar to master production scheduling in a manufacturing context. Scheduling the PO due dates should start with a just-in-time approach and then be adjusted based on the available receiving capacity at the DC. Without such adjustment, POs could all arrive on the same day of the week, causing DC receiving bottlenecks. The goal of MPRS is to align deliveries with DC receiving capacity. The PO due dates can vary greatly based on lead time, ranging from weeks to months in advance. Note that at the MPRS level, the PO due dates are at a daily rather than hours or minutes level.
2. Detailed Level: Detailed Receiving Scheduling. This usually addresses product receiving within the next few days, done shortly before a truck departs its point of origin. The detailed receiving schedule will specify the time slot when a truck should be received, down to the hour or minute level. Note that an MPRS indicates the total number of deliveries on a given day, and detailed receiving scheduling takes that as an input and schedules the precise timing of the day's deliveries. Both processes are required to avoid DC bottlenecks.

Currently, most companies do not put much effort into MPRS. They place POs based on a just-in-time approach without considering the potential DC receiving constraints. At best, some companies rely on a manually created MPRS. However, fluctuating product demand and holidays make leveling the receipts during the week a time-consuming process that can result in a suboptimal MPRS. In the absence of a proper MPRS process, companies are forced to dedicate excessive time and resources to adjust delivery schedules. That can be costly, as they may have to ask truck drivers to delay or expedite shipments depending on the dock schedule.

A robust methodology for optimally scheduling inbound deliveries at the time of placing POs promises to improve resource utilization and enhance overall DC throughput. It must consider various factors, such as vendor shipping calendars, DC receiving calendars, shipment sizes, and the inherent unpredictability of transportation. The result is a MPRS that is not just feasible but also economically optimal.

This article presents a novel algorithm developed by New Horizon Soft, LLC, a Boston-based supply chain software vendor, using data from a major U.S. quick-service restaurant chain. The chain is using this algorithm for generating a MPRS for its DC operations. The algorithm is described in detail, and the results of a Monte Carlo simulation based on historical shipment data are presented. This is followed by a discussion of its effectiveness, an extension to the algorithm, and its strengths and weaknesses. While the development of this algorithm was prompted by the needs of a restaurant chain, it addresses a problem that is broadly relevant to any business with an interest in reducing DC receiving bottlenecks and increasing cost efficiency.

As illustrated in the next section, Prior Research, there has been much research on related aspects of DC operations in the areas of receiving, transportation, and replenishment, but little research focusing on MPRS. The motivation for this article is to stimulate further research into this important but relatively neglected area of supply chain management.

PRIOR RESEARCH

Some research has been conducted on inbound shipments scheduling. However, most research on DCs focuses on the put-away and order-picking processes because they have a more direct impact on traditional measures of DC performance and are the most labor-intensive processes (Boeve, 2016). Additionally, Jackson (2005) states, "There is also an organizational belief that outbound is always more important than inbound since it is the customer-facing part of the warehouse" (p. 10). As a result, insufficient research has been conducted specifically on the inbound vendor receiving

process, a fact that became apparent to the restaurant chain that is the subject of this case study and that required more efficient and automated scheduling. This is especially true in the area of MPRS.

The inbound vendor receiving process starts with demand planning, in which planners forecast how many goods will be sold. Based on the resulting demand schedule, material requirements planning (MRP) logic will generate purchase quantities using multiple approaches, as outlined in Monczka et al. (2020). Methodologies include economic order quantity, reorder point, just-in-time, kanban, etc. Nagpal et al. (2022) proposed an innovative inventory optimization model for short-life cycle products that is suitable for the high-tech industry. Thurman (2021) assessed different inventory replenishment strategies for Target Corporation. Hwang (2009) studied a dynamic lot sizing vendor-managed inventory warehouse model to illustrate the importance of the minimum replenishment quantity. Pandey (2022) outlined a two-step approach that first generates a so-called "pristine order" to meet just-in-time needs. Then in the second step, different constraints, including product availability and truck load needs, are used to rationalize the "pristine order."

Many papers address the DC operations aspects of the problem. Cochran and Ramanujam (2006) analyzed optimal combinations of third-party logistics providers, container sizes, packaging (such as pallets and slip sheets), and additional services to minimize costs. Traditional dock scheduling approaches have been studied, and more recently there has been research on more advanced algorithms such as genetic algorithms (Fong et al., 2013). Similarly, truck optimization models using mixed-integer linear programming have been developed for DCs with a mixed service-mode dock area (Correa Issi et al., 2020). Some models also take into account the different types of vehicles with different capacities in DC cross docking and minimize operation time, lateness, and earliness of product delivery (Mohtashami, 2020). However, dock scheduling focuses on the short term, since each truck's exact time slot must be determined and it is difficult to do so weeks and months ahead.

From a transportation perspective, books like that of Novack et al. (2018) provide a good summary of transportation management. The simplex method has been used in solving some transportation optimization problems for years (Dantzig, 1951). Boeve (2016) has introduced improvements to the inbound system based on available space, staff capacity, and variability in vendor deliveries, and Jackson (2005) has analyzed the impact of flexible staffing and work schedules. There are other publications focused on truck load optimization such as Morabito et al. (2000) and Alonso et al. (2019). Calabrò et al. (2020) focuses on minimizing truck travel distance by using an ant-colony simulation-based optimization for finding routes, and Wang and Chen (2019) have developed a time-indexed integer linear programming model to minimize truck transportation time in supplier parks. Yang et al. (2023) proposed an integrated deep reinforcement learning-based logistics management model (DELLMM) to improve the logistic distribution.

While much research has been conducted on inbound receiving, transportation, replenishment, and short-term receipt scheduling, as illustrated in the above discussion, it only tangentially addresses the longer-term receipt scheduling process we have named MPRS. Receipt scheduling at the time of placing POs remains an underexplored area, and its time-consuming nature remains unaddressed.

DETAILED PROBLEM STATEMENT

This paper focuses on inbound delivery scheduling, for which we've introduced the concept of MPRS. The goal is to schedule vendor delivery dates when POs are placed so that the daily volume of inventory received at the DC will be as consistent as possible for efficient use of DC receiving resources.

Many companies use calendars with designated receiving days of the week for each vendor during regular (non-holiday) weeks. Usually, the number of days available to the vendor depends on the quantity of goods being delivered, the vendor's workdays, and the receiving DC's workdays. Buyers place POs with the vendors several weeks ahead of time and communicate the desired delivery dates.

However, typical PO generation logic does not consider delivery optimization, and there are no readily available algorithms to automatically generate a vendor delivery calendar factoring in the DC's inbound loading resources. Furthermore, several major challenges arise from manual scheduling:

1. As the demand for goods changes by season, delivery days must be rescheduled to better level regular weeks.
2. Many vendors and DCs are closed on certain holidays such as Thanksgiving and July 4th, restricting the delivery calendar. Thus, planners must further readjust the MPRS to level deliveries and supply demand for the week while adhering as much as possible to the new schedule and vendors' usual designated delivery days. These constraints often leave some days overloaded. Additionally, since demand varies by holiday and year, one holiday week's schedule is unlikely to work for future holidays, so each requires a new optimized schedule.

These time-consuming issues lead to planners spending excessive amounts of time creating DC receiving schedules for each vendor, with most weeks requiring up to 12 additional hours for the lead planner at the aforementioned restaurant chain while still yielding a suboptimal MPRS schedule. The goal of the described algorithm is to give planners a tool for creating POs with optimal due dates which will level daily receiving calendars for the DC, saving time and money.

THE LEVELING ALGORITHM

To replace the existing manual process, the authors developed a finite scheduling algorithm to recommend a MPRS for vendor deliveries. The algorithm starts by calculating weekly vendor shipments in terms of pallets, weight, and cube and then converts this to the number of trucks each week for each vendor. It then categorizes the vendors into large, medium, and small based on the number of trucks and then calculates possible DC receiving days based on each vendor's available shipping days, transit time, and DC workdays. The algorithm aims to balance the distribution center's workload by spreading out deliveries, starting with large vendors, and then fitting in medium and small vendor deliveries within the constraints of daily receiving capacities. Through iterative refinements, it reduces daily intake variance to ensure the most efficient receiving schedule.

The key aspects of the algorithm are described below.

Input Data and Variables

The variables and input data for the algorithm are summarized in Table 1 below. These will be described in more detail in subsequent sections.

Algorithm Flow Chart

A high-level process flow for the algorithm is outlined in Figure 1 below, followed by a step-by-step description.

Below are descriptions of each step:

1. **Read In Data and User Inputs:** This includes the data listed in the "Input Data" column of Table 1. Some are user-defined parameters and others are item attributes, planned purchases, and open purchase orders.
2. **Calculate Item Pallets, Weight, and Cube for Each Week:** This calculates the total amount of a vendor's item needed in terms of pallets, weight, and/or cube for each week using planned purchases and already-purchased orders data.
3. **Calculate Vendor Trucks for Each Week:** This converts the pallets, weight, and/or cube for each item into the weekly number of trucks for each vendor using their truck conversions. A truck
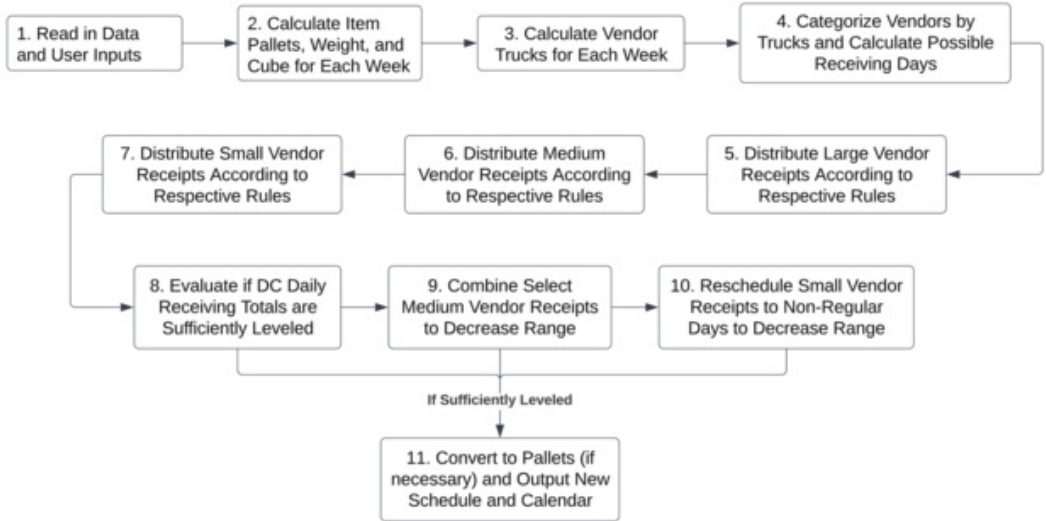
Figure 1. Leveling Algorithm Flow Chart

Table 1. Leveling Algorithm Variables and Input Data

Variable	Description	Input Data
$D_{receive}$	Date that inventory is received at the DC	Number of Weeks w to Schedule
D_{ready}	Date that inventory is ready to leave DC for outbound delivery	Starting Date for Schedule
$t_{post\,lt}$	Post-processing lead time, or days to process inventory for outbound delivery	Minimum Trucks to be considered a Large Vendor
$P_{x\,total}$	The total number of pallets for an item for week x	Minimum Trucks to be considered a Medium Vendor
$P_{y\,planned}$	The planned number of pallets to purchase for an item y days from the start date	Vendor Transit Time and Post-Processing Lead Time
$P_{y\,bought}$	The number of pallets already purchased for an item y days from the start date	Vendor and DC Holidays
$C_{x\,total}$	The total number of cases for an item for Week x ($x = 0$ is the first week)	Vendor Delivery Calendar for Regular Weeks
$T_{x\,pallet}$	The calculated number of trucks based on pallet capacity for an item for Week x	Planned Pallets/Weight/Cube of an Item to Purchase for Next $w+1$ Weeks
$T_{x\,weight}$	The calculated number of trucks based on weight capacity for an item for Week x	Pallets/Weight/Cube of an Item that are Already Purchased for Next $w+1$ Weeks
$T_{x\,cube}$	The calculated number of trucks based on cube capacity for an item for Week x	Vendor Truck Capacities and Item Weight and Cube per Case
$V_{x\,p\text{-}to\text{-}t}$	The average number of pallets per truck for a vendor for Week x	

corresponds to a PO for which all the products inside the truck will arrive together. Revising the delivery date of the truck is the same as revising the PO due date.

4. **Categorize Vendors by Trucks and Calculate Possible Receiving Days:** This categorizes the vendors into large, medium, and small vendors based on the number of trucks needed in a week and then calculates possible DC receiving days based on each vendor's available shipping days, transit time, and DC workdays.

5. **Distribute Large Vendor Receipts According to Respective Rules:** For large vendors, we generally spread the deliveries over every possible day of the week to reduce overloading and inventory on hand.
6. **Distribute Medium Vendor Receipts According to Respective Rules:** For medium vendors, we try to arrange deliveries on days with fewer trucks so the DC's total received trucks for each day of the week can be leveled.
7. **Distribute Small Vendor Receipts According to Respective Rules:** Small vendors' deliveries can also be moved around to days with currently fewer trucks to level the daily truck distribution in a week.
8. **Evaluate if DC Daily Receiving Totals Are Sufficiently Leveled:** This checks if the DC daily receiving totals are leveled after steps 2 through 7. This is possible due to how we define large, medium, and small vendors.
9. **Combine Select Medium Vendor Receipts to Decrease Range:** This adjusts certain medium vendors by combining multiple deliveries into one to quickly reduce the variation in daily trucks to be received by a DC.
10. **Reschedule Small Vendor Receipts to Non-Regular Days to Decrease Range:** This makes further adjustments to small vendors (if necessary) by having them deliver on days outside their regular schedule to further reduce the variation in the number of trucks each day for the DC.
11. **Convert to Pallets (if Necessary) and Output New Schedule and Calendar:** This converts shipment quantities into pallets if there is a pallet constraint and publishes a new schedule and calendar for vendors and the DC.

Assumptions and Justifications

For the leveling process to balance the receiving schedule (MPRS) as much as possible, two assumptions were made:

Assumption 1: A vendor's deliveries may be distributed over any day of the week as long as it is available on the regular calendar and not a holiday for the DC or the vendor (taking into account transit time).
  ◦ Justification: Though this may mean that some products may arrive later than their original date, safety stock can cover any deficit during this short transition period. This approach is preferable to allowing deliveries to be pulled forward. Such a policy would lead to overstocking and congestion earlier in the week (from receiving and processing most of the week's demand) and to underutilizing receiving resources later in the week.

Assumption 2: When converting deliveries to truckloads, they will be rounded to the nearest 0.1, 0.25, or 0.33 of a truck (which covers important portions of a truck: 0.2, 0.5, 0.67, and 0.75) unless the receiving amount is so small that rounding leads to extreme inaccuracy.
  ◦ Justification: Rounding to the nearest 0.1, 0.25, or 0.33 of a truck easily allows planners to know how often a particular truck should arrive to deliver the order. For example, 1.1 trucks means two trucks for every ten orders (one truck for the rest of the nine orders). However, for extremely small orders, rounding is avoided since it can lead to over-planning of trucks. For example, we will not round 0.05 truck to 0.1 truck for the restaurant chain's DCs since that would make deliveries twice as frequent (a truck every 10 weeks rather than every 20 weeks).

Detailed Explanation of the Algorithm

The following is a detailed description of each of the 11 process steps outlined in the Algorithm Flow Chart section. Each major section below is numbered 1 through 11 accordingly.

Read In Data and User Inputs

This includes the data listed in the "Input Data" column of Table 1 in the Input Data and Variables section. Some are user-defined parameters and others are item attributes, planned purchases, and open purchase orders.

Calculate Item Pallets, Weight, and Cube for Each Week

The planned purchase and already-purchased order data are based on the date that the inventory will be available and ready to deliver to customers. Since the date of arrival (or receipt) at the DC is desired, it is determined by Equation 1:

$$D_{receive} = D_{ready} - t_{post\,lt} \tag{1}$$

where $D_{receive}$ is the date of receiving inventory, D_{ready} is the date when inventory is ready for outbound delivery, and $t_{post\,lt}$ is the number of days to process after the PO arrives at the DC.

Based on the inputted number of weeks w to schedule, the next w full weeks (or $7w - 1$) days of planned purchase, which is based on the replenishment method (e.g., EOQ, reorder point), and already-purchased order arrival data are added together to get the daily total pallets, weight, or cube (volume). The daily totals are then summed up to produce the weekly totals for the item. Here we use pallets as an example for Equation 2:

For each week x:

$$P_{x\,total} = \sum_{y=7x}^{7x+6} \left(P_{y\,plan} + P_{y\,bought} \right) \tag{2}$$

where $P_{x\,total}$ is the total pallets for week x, and $P_{y\,plan}$ and $P_{y\,bought}$ are the planned purchase pallets and already-purchased pallets (not yet received) for the yth day after the starting date. One could use the same formula to calculate the total weight and total cube for each item, vendor, and week.

Calculate Vendor Trucks for Each Week

Though this quick-service restaurant chain operates using pallets as units, they need to level the number of deliveries during the week, which depends on the number of trucks. So, the number of trucks for each item that a vendor supplies in the week must be determined. Vendors fill trucks up to capacity based on pallets, weight, or cube, but most only use one or two of these metrics. So, to get the number of trucks that fulfill all of the truck capacity conditions, we aggregate each item's weight, cube, and pallet and use such to calculate the number of trucks. The highest of the calculated number of trucks by each dimension is used to get the number of trucks for the vendor that fulfills all constraints. Sample data for a week is shown in Table 2.

Explanation of the table:

- The weekly required pallet number is 18
- 33 cases in one pallet
- A truck can hold a maximum (max from here on out) of 40 pallets and a max of 44,500 lbs, but there is no limitation on cube (meaning no constraint on volume)
- A case is 25.65 lbs and 1.15 cubic feet (cube)

Table 2. Sample Item Data

Weekly Pallets $P_{0\,total}$	Cases per Pallet C_{pallet}	Max Pallets Max_{pallet}	Max Weight (lbs) Max_{weight}	Max Cube (ft³) Max_{cube}	Case Weight (lbs) C_{weight}	Case Cube (ft³) C_{cube}
18	33	40	44500	*None*	25.65	1.15

The calculated number of trucks based on pallet, weight, and cube are shown in Equations 3-6 below:

$$C_{0\,total} = P_{0\,total} \times C_{pallet} = 33 \times 18 = 594 \text{ cases} \tag{3}$$

$$T_{0\,pallet} = \frac{P_{0\,total}}{Max_{pallet}} = \frac{18}{40} = 0.45 \text{ truck} \tag{4}$$

$$T_{0\,weight} = \frac{C_{0\,total} \times C_{weight}}{Max_{weight}} = \frac{594 \times 25.65}{44500} \approx 0.3424 \text{ truck} \tag{5}$$

$$T_{0\,cube} = \frac{C_{0\,total} \times C_{cube}}{Max_{cube}} = \frac{594 \times 1.15}{999999} \approx 0 \text{ trucks} \tag{6}$$

where $C_{0\,total}$ is the number of cases of an item for week 0 (the first full week after the inputted starting date), and $T_{0\,pallet}$, $T_{0\,weight}$, and $T_{0\,cube}$ are the number of trucks necessary for week 0 based on the max pallet, max weight, and max cube criteria, respectively.

Since the vendor in the sample does not use max cube as a constraint, an arbitrarily large number (999,999) was substituted, so total trucks based on cube is essentially zero: $T_{0\,cube} \approx 0$ (therefore, cube will not be considered a constraint, as will be shown below). When we buy multiple items from the vendor, we can aggregate by number of pallets, weight, and cube and then convert such into the number of trucks using the same formula.

The number of trucks for this vendor for this week if we are only buying this item is calculated using Equation 7:

$$max\left(T_{0\,pallet}, T_{0\,weight}, T_{0\,cube}\right) = T_{0\,pallet} = 0.45 \text{ truck} \tag{7}$$

Categorize Vendors by Trucks and Calculate Possible Receiving Days

After each vendor's total trucks for each week are calculated, they must be distributed over the week. The restaurant chain's DCs are closed on Sundays, so vendors can only deliver Monday through Saturday. Since vendors have a wide range of truck quantities, they are split into three groups to determine how many receiving days they should have during the week as follows:

1. Large vendors (greater than the user-inputted minimum trucks for large vendors): these vendors, on most days, are delivering at least a full truckload. So, they should have a receiving slot every available day to spread out their large deliveries. If we use four trucks per week as a criterion, the vendor will deliver more than half of the days in the week.
2. Medium vendors (greater than the user-inputted minimum trucks for medium vendors): these vendors have fewer trucks, so receiving them every day is not necessary. For this range, two to

three days per week are sufficient so each time at least most of a full truck is received. A criterion between 1.5 and four trucks can be used for medium vendors.

3. Small vendors (smaller than the minimum trucks for medium vendors): these vendors have the smallest quantities, so only one receiving day per week is sufficient.

However, to minimize the change from the regular calendar, vendors are only allowed to deliver on their regular available delivery days and non-holidays. The new schedule accounts for these by copying the regular receiving schedule for the vendor and then overwriting days that are holidays. However, vendor holidays must be converted to the DC receiving days based on their transit times, since if a vendor is closed, it affects the day that the truck would arrive at the DC (if the vendor is closed on July 4th and has a transit time of 1 day, a July 5th receipt at the DC is impossible). In the new receiving schedule, "1" indicates a possible receipt, "H" indicates a DC or vendor holiday (adjusted for transit time), and "None" indicates an unavailable day for the vendor. An example is shown in Table 3 below.

●   In Table 3 above, for vendor ID 1, the regular delivery schedule is Tuesday, Wednesday, Thursday, and Friday for the week of 7/2/2023. Saturday and Monday are not available, and the DC is closed on Sunday (DC holiday).
●   Now the vendor has a holiday on July 4th, so the DC cannot receive the vendor's delivery on July 5th.
●   The new schedule with holidays is Tuesday, Thursday, and Friday with Wednesday as a holiday.
●   Future DC holidays on 11/23 and 12/25 are irrelevant for that week.

Distribute Large Vendor Receipts According to Respective Rules

We designed each vendor group with different receiving distribution methods since they have varying levels of freedom in receiving days. For example, small vendors typically have the most freedom because they usually have multiple days to choose from but only need to deliver on one day.

For large vendors, we only look at the expected receipts by each vendor. We use the following rules, with examples for each step shown in Table 4 and Table 5:

1. Obtain the number of days of demand the vendor's trucks should cover for each available delivery day. This is done by creating a schedule for this week and next week and finding the number of workdays from the first possible receiving day of the current week to the first possible receiving day of the next week (highlighted in green) based on transit time.

From Table 4, we have used a transit time of 1 day. The first receipt of the current week is July 4th (vendor shipping out on Monday, July 3rd, and arriving on July 4th at the DC). The next available date for the following week is July 11th, but the DC is closed on Sunday, July 9th, which means the current week's deliveries need to cover 6 workdays' demand (July 4th till July 10th, 2023 except Sunday, July 9th, as highlighted in green on those dates in Table 4).

2. Divide the weekly trucks by the total number of demand days to cover to get the daily amount to deliver. Each workday's receiving amount is the daily amount multiplied by the number of DC workdays that day's products must cover.

If the vendor has 4 trucks this week, the daily amount is ⅔ of a truck. See the results in Table 5. The receipt on July 4th must cover July 4th as well as July 5th since there is no receipt on that day, so it has double the daily amount (weekly truck is 4, daily is 4/6 = 0.67; doubling that is 1.33). Similarly, since July 7th covers July 8th and July 10th, the DC will receive three times the daily amount

Table 3. Example Schedule Setup
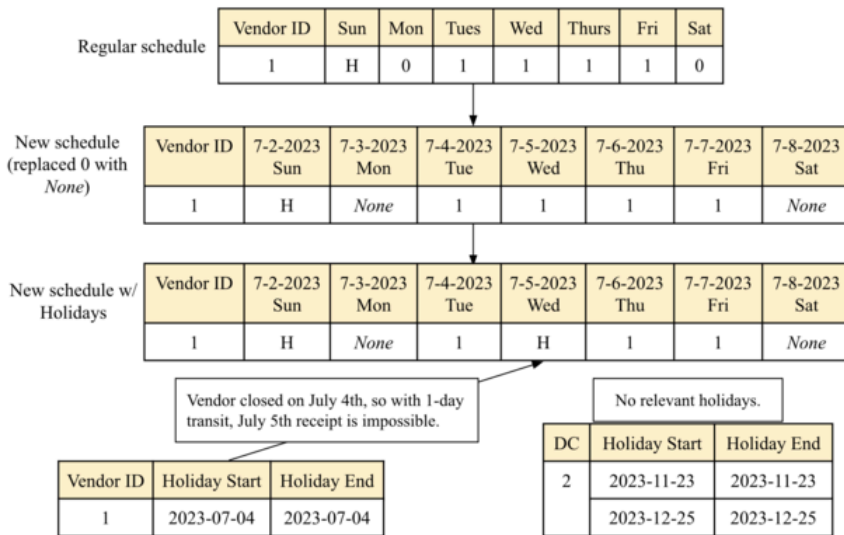
then (0.67*3 = 2). If necessary, the receiving amount results are then rounded to the nearest 0.25, 0.33, or 0.1 of a truck (see the Assumptions and Justifications section) to maintain predictable, semi-regular deliveries for additional trucks without deviating much from the forecasted demand.

Distribute Medium Vendor Receipts According to Respective Rules

We assigned medium vendors with the most rules since they have multiple deliveries that should be spaced apart to avoid temporary overstocking. Additionally, because the number of days the medium vendors deliver depends on the number of trucks and available days, medium vendors follow different rules. Examples for the four cases are shown below, and similar to the large vendors, all values are rounded. We are marking "1" as placeholders with the distributed deliveries.

We constrain the number of receiving days to two or three for the week based on the range of the user input to classify medium vendors. In the "medium vendor" range of trucks, the lower half will have two days and the upper half will have three days. For example, if we assign a range from 1.5 to four trucks during a week to be a medium vendor, any vendors with 2.75 trucks ((1.5+4)/2) or above will be assigned three receiving days, and any with fewer trucks will be assigned two days.

Case 1: Sufficient trucks to have three receiving days, and exactly three possible receiving days after combining constraints from the regular schedule and holidays (all open days will be used). An example is shown in Table 6 where only Tuesday, Wednesday, and Friday are deliverable.

Split the receiving amounts evenly across the three days, as it is assumed the user may lower the minimum large-vendor criteria if distributing the vendor's trucks based on the days of demand to cover is desired. Table 6 shows the results. Each day (July 4th, July 5th, and July 7th) gets an equal number of trucks (1.2 trucks). However, if the three days are all consecutive, follow the large vendor algorithm to minimize overstocking during the first two days by using the days of demand to cover approach.

Case 2: Sufficient trucks to have three receiving days, but more than three possible receiving days (only three days will be chosen). To determine which three days to deliver, we need to look at the total number of trucks by day by DC which has been assigned so far. This gives us some room

Table 4. Sample Days-to-Cover Determination

Current Week

Vendor ID	7-2-2023 Sun	7-3-2023 Mon	7-4-2023 Tue	7-5-2023 Wed	7-6-2023 Thu	7-7-2023 Fri	7-8-2023 Sat
1	H	*None*	1	H	1	1	*None*

Next Week

Vendor ID	7-9-2023 Sun	7-10-2023 Mon	7-11-2023 Tue	7-12-2023 Wed	7-13-2023 Thu	7-14-2023 Fri	7-15-2023 Sat
1	H	*None*	1	1	1	1	*None*

Days to Cover = 6

Table 5. Sample Large Vendor Result

Vendor Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
4	H	*None*	1.33	H	0.67	2	*None*

to move the trucks to the lowest delivery days to balance out the truck load distribution across the week. Examples are shown in Tables 7 and 8. They follow the three steps below:

1. Sum up at the DC level the total trucks for each day in a week that has been assigned so far. From the available receiving days, choose the one currently with the fewest trucks delivering. See Table 7.

Tuesday, July 4th, highlighted in green below in Table 8, has the fewest trucks, so we assign it as a day to deliver:

2. Check how many days the DC is available for the week:
   a. If the DC is open fewer than five days of the week, choose the next two lowest receiving days, split receipt amounts evenly among the three days, and move to the next vendor.
   b. Otherwise, choose any open days that are two or four days away from the first receiving date to spread deliveries across the week. If both of those days are available, divide the deliveries evenly and finish.
   c. If neither of these days are found, choose the earliest day in the week that is at least two days away from the first receipt (can be before or after). This ensures that two dates will be chosen. Table 8 is updated below to include the second selected receiving day (in green) as shown in Table 9 (July 7th is selected).
3. Finally, choose any days that are more than one day away from both deliveries.
   a. If this yields one additional date, distribute trucks evenly over the three days.
   b. If this returns multiple available dates, select only the date with the lowest number of trucks and distribute the trucks evenly over the three days. Picking the lowest number of trucks will help balance the overall truck delivery among days in the week.
   c. Lastly, if this does not return any dates, select the remaining day with the fewest trucks and distribute evenly. Table 9 is updated below further to include the last selected receiving day as shown in Table 10.

Table 6. Sample Medium Vendor Three-Day Result (Each Day Gets 3.5/3 = 1.2 Trucks)

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri
2	3.5	H	None	1.2	1.2	H	1.2

Table 7. Sample DC Daily Trucks (for Medium Vendors)

	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
Trucks	H	15	13	14.8	15.5	14.6	17

This example does not have any deliveries that are more than one day away from other deliveries, so using Table 7, the remaining day with the least number of trucks is selected, which is July 3rd (highlighted green).

Case 3: Exactly two possible receiving days regardless of the number of trucks (both days are used).

If the number of trucks is only sufficient for two receiving days, split the deliveries evenly (similar to Table 6), following the same reasoning in Case 1. Otherwise, there are enough trucks among the two days to follow the same procedure as a large vendor, so distribute the receipt amounts based on the number of days each receipt must cover (similar to large vendors as in Table 5).

Case 4: Sufficient trucks for only two receiving days, but more than two possible receiving days (only two will be used). To determine which two days to deliver, we need to look at the total number of trucks by day by DC which has been assigned so far (similar to Case 2). Follow the below two-step process:

1. Select the available day with the fewest trucks. Sample data is shown in Table 11 below where Tuesday is a holiday, and calculations are based on sample DC trucks from Table 7. Of the available dates, July 5th has the fewest trucks (14.8), and therefore it is highlighted in green.
2. Select the available day or days farthest away from the first receiving date. For a six-day working schedule, three days away is optimal to prevent overstocking of goods if two days are too close. If multiple days are selected, choose the one with the current least number of total trucks. Then, distribute the deliveries evenly. In this example, July 8th is picked, as shown in Table 12, since it is the available day farthest away from July 5th in the week.

Distribute Small Vendor Receipts According to Respective Rules

We assign small vendors the last since they only have one receipt a week. Below is the logic we used:

Sum up at the DC level the total trucks for each day in the week. Deliver on the day of the week available to the vendor with the fewest number of trucks. Results are shown in Tables 13 and 14 below.

Of the days available in Table 13, July 4th has the fewest trucks (18). Therefore, assign Tuesday as the delivery date for this small vendor (see Table 14).

Evaluate if DC Daily Receiving Totals are Sufficiently Leveled

Although both the distribution algorithms for medium and small vendors account for how many trucks the DC is already receiving each day and try to balance it out, delivery restrictions will likely

Table 8. Sample Medium Vendor Three-Day Schedule

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
3	3.5	H	1	1	H	1	1	*None*

Table 9. Updated Medium Vendor Schedule After Step 2

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
3	3.5	H	1	1	H	1	1	*None*

Table 10. Final Updated Medium Vendor Schedule

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
3	3.5	H	1.2	1.2	H	0	1.2	*None*

cause the DC's receipt schedule to remain unleveled. Since large vendors have little flexibility due to how many trucks they have, only medium and small vendors should have their deliveries modified. Small vendors are entirely based on DC trucks before being assigned, so any further improvements would mean changing the vendor's regular schedule, which should only be a last resort. Medium vendors also have more trucks, so adjusting their deliveries first has the most impact.

Combine Select Medium Vendor Receipts to Decrease Range

Since medium vendor receipts are spread over multiple days, one way to reduce the variation of DC trucks is to combine a vendor's receipts from multiple days and move them to the least busy available DC day. However, there is no guarantee that each medium vendor adjustment will decrease the range of truck amounts since many cannot be received on both the busiest and the least busy days of the week.

Therefore, the most intuitive and simple strategy is to only combine receipts for vendors that deliver on the week's least busy day, to move trucks there, or on the busiest day, to move trucks away. While this means that certain vendors will not have leveled deliveries, the overall week will be more balanced, which is a higher priority. The goal is to level all DC workdays to within one truck for the most optimized solution, and the following two steps are used until all days are leveled or there are no more medium vendors to adjust:

1. Select medium vendors with deliveries on the current busiest or least busy day of the week (not including DC holidays). A sample DC's aggregated daily trucks are shown in Table 15, and a sample vendor's schedule is shown in Table 16.

July 3rd currently has the most deliveries, and the vendor is delivering on that date (highlighted in red in Table 16), so it fulfills the conditions above.

Move all deliveries to the available day with the fewest number of DC trucks. If the vendor is delivering on the busiest day of the week, those trucks get moved to a less busy day, and if the vendor is delivering on the least busy day of the week, then that day will gain more trucks. In either case, the variation of daily DC truck totals (ignoring holidays) will almost always decrease, either by increasing the minimum or decreasing the maximum. Tables 15 and 16 are updated below to Table 17.

Table 11. Sample Medium Vendor Two-Day Schedule

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
4	2.5	H	*None*	H	1	1	*None*	1

Table 12. Updated Medium Vendor Two-Day Schedule

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
4	2.5	H	*None*	H	1.25	0	*None*	1.25

Table 13. Sample DC Daily Trucks Aggregated by Day

	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
Trucks	H	25.25	18	20.5	21	22	21.67

Table 14. Sample Small Vendor Result

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
5	1	H	*None*	1	H	0	0	*None*

Out of the two receiving days besides July 3rd (July 5th and July 7th), July 7th has fewer trucks, so the two trucks from July 3rd and July 5th are added to July 7th to decrease the truck range by one. Thus, July 3rd and July 5th each lose a truck, and July 7th gains two trucks.

Reschedule Small Vendor Receipts to Non-Regular Days to Decrease Range

As mentioned above, since small vendors are scheduled to avoid busier days, the only way to further decrease the daily truck variation is to move their deliveries to days outside their normal schedule. Often, since the medium vendors create larger net changes in the DC truck range, a few small vendors need to be modified to decrease the range to within one truck. Thus, small vendors that can deliver on the least busy day (as long as they are not on holiday) are selected. To minimize the number of changes, smaller vendors with more trucks are modified first with the following rule:

Move the current receipt to the day in the entire work week with the fewest number of trucks. An example is shown below in Table 18.

Since before the adjustment July 4th has the fewest trucks (23.75), the small vendor's receipt is moved from July 6th to July 4th to decrease the truck range over the week by one.

Convert to Pallets (if Necessary) and Output New Schedule and Calendar

Finally, after all DC days are leveled to within one truck for each week, the receipt schedule is converted to pallets based on a pallet-to-truck conversion. Assuming we want to display all purchase order data in pallets, an average pallet-to-truck conversion can be established for each vendor for Week x with Equation 8:

$$V_{x p\_to\_t} = \frac{\Sigma P_{x\,total}}{\Sigma max\left(T_{x\,pallet},\ T_{x\,weight},\ T_{x\,cube}\right)} \tag{8}$$

where $V_{x p-to-t}$ is the vendor's average pallets per truck, and $\Sigma P_{x\,total}$ and $\Sigma max\left(T_{x\,pallet},\ T_{x\,weight},\ T_{x\,cube}\right)$ are the total pallets and trucks for all of the vendor's items, respectively.

Table 15. Sample DC Daily Trucks of the Week (Before Medium Vendor Adjustment)

	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
Trucks	H	30.25	22.5	26.5	26.7	24.33	27.67

Table 16. Sample Medium Vendor Adjustment

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
6	3	H	1	H	1	0	1	*None*

Table 17. Adjusted Medium Vendor Schedule

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
6	3	H	0	H	0	0	3	*None*

Additionally, another calendar consisting of ones and zeros to only denote receiving and non-receiving days, respectively, was created to easily see when to receive each vendor's inventory.

ANALYSIS AND DISCUSSION OF MODEL PERFORMANCE

Monte Carlo Simulation

Because the quick-service chain's order quantities vary depending on the week, it is important for the new algorithm to be robust and be able to level deliveries even when each vendor's number of trucks changes. As a result, a Monte Carlo simulation of 1,000 trials was tested by restricting vendors to deliver only on days available in the new receiving calendar generated by the algorithm. To add variability, each vendor's number of trucks varied between 90% to 110% of the original value. The new calendar's robustness was first tested on a regular week with no holidays. The truck and pallet ranges for the DC week and the maximum and average percent variation from the mean are shown in Figure 2. Ranges are defined as shown in Equation 9:

$$\text{Range} = max(DC_{day}) - Min(DC_{day}) \text{ during the week} \tag{9}$$

where DC_{day} is the number of trucks or pallets that the DC needs to receive during any day of the week (except holidays).

Percent variation from the mean was calculated as the absolute maximum difference from the mean pallets or trucks per workday, divided by the mean pallets or trucks, as shown in Equation 10:

$$\% \text{ variation} = \frac{max(|DC_{day} - \mu_{DC}|)}{\mu_{DC}} \tag{10}$$

where μ_{DC} is the mean DC daily truck or pallet total during the workdays of the DC.

In Figure 2, the X-axis is the total number of pallets or trucks that need to be received during the week, and the Y-axis is the range for each simulation. We can see the maximum range is approximately 105 pallets or 4.6 trucks, and the mean range is about 75 pallets or three trucks. Given that the average number of pallets is about 3,600 for the week, or 600 pallets per workday, and the average number of trucks is about 150, or 25 trucks per workday, the change in daily variation is fairly controllable.

Table 18. Sample Small Vendor Adjustment

Thus, the new calendar is reasonably robust, as a random variation of 10% in vendor order quantities still produces a schedule with a maximum variation of 11-12% and a mean variation of 7% from the average. However, since the receiving calendar for a regular week has fewer restrictions, the algorithm was then tested on the July 4th holiday week (using purchasing and holiday data for that week) with the same amount of variability, leading to the following results in Figure 3:

The ranges are substantially higher, as the pallet variation has a maximum of about 150 and a median of about 105, and the truck variation has a maximum of almost 6 and a mean of about 3.5. Additionally, the average daily receipt totals are about 580 pallets or 24 trucks.

While the percent variations are noticeably higher in this situation, this increase is to be expected since the holidays are inherently more disorganized than regular weeks. However, the new calendar still produces reasonably robust results, as each day differs from the mean by a maximum of 16% and a mean of 11.5% for pallets and a maximum of 13% and a mean of 8% for trucks out of 1,000 simulations.

DC Receiving Capacity Extension

While the base algorithm described above for MPRS can successfully optimize DC receiving schedules by leveling the number of trucks delivering each day, an extension to account for a DC's maximum receiving capacity per day may be considered for easier applicability to other companies that may require different optimization constraints. This extension was also used in the case of the quick-service restaurant chain, and the methods for achieving the constraint are described below.

Adding Truck Limits for Certain Days

While evenly distributing truckloads across the week is beneficial, some companies may prefer having a cap on the maximum number of trucks to receive on particular days, such as days when there are fewer DC staff or when employees don't work full shifts. To account for this condition, an additional process should be added to the distribution of large, medium, and small vendors.

Figure 2. Regular Week Monte Carlo Simulation
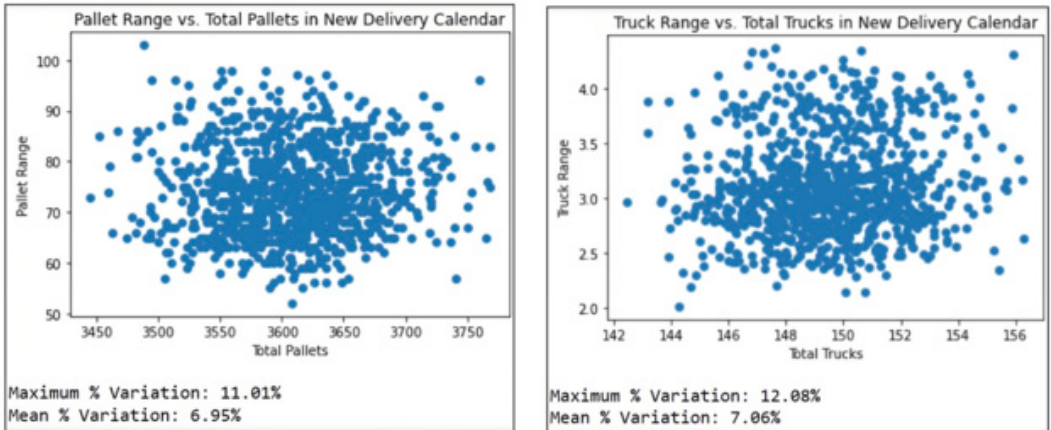
To first ensure that vendors with only one delivery day for a week can deliver, the distribution process should begin by automatically assigning these vendors to deliver on their available day. This way, they do not run the risk of being unable to deliver if other vendors occupy that day before they do. Even if a day's maximum capacity is set extremely low such that it is impossible to remain below the limit, this distribution should be carried out since it is preferable to see which vendors are unable to move rather than ignoring the vendor and not distributing their trucks at all. It is therefore important that these limits be reasonably chosen based on the number of trucks that must be delivered on that day.

For the rest of the vendors, the truck distribution logic for MPRS remains the same as previously described except that another table of the remaining capacity for each day must be created. The table's values for each day of the week should be set to the maximum receiving capacity if present and "*None*" for days without inputted capacities, as in the example below in Table 19:

Explanation of the table:

- Sunday is not a workday, so its daily capacity is set to zero. Using a DC custom limit table also allows companies to mark which days DCs are open, making the algorithm more applicable to different availabilities.
- Tuesday, Thursday, and Friday all have maximum capacities set at a specific number of trucks, while Monday, Wednesday, and Saturday have no limit imposed.

Truck Distributions With DC Limits

To make sure there is enough capacity to receive a vendor's trucks, the distribution algorithm should be updated with the following procedure:

1. Go through the procedures outlined in the section Detailed Explanation of the Algorithm to determine the best dates and how many trucks to deliver on each day. Sample DC truck totals at a point during the algorithm are shown in Table 20, and a sample large vendor distribution is shown in Table 21.
   a. This vendor is assumed to not have holidays this week or the following week, so the first delivery date the following week is July 11th, meaning there are 7 days to cover for the week of July 2nd.

Figure 3. Holiday Week Monte-Carlo Simulation
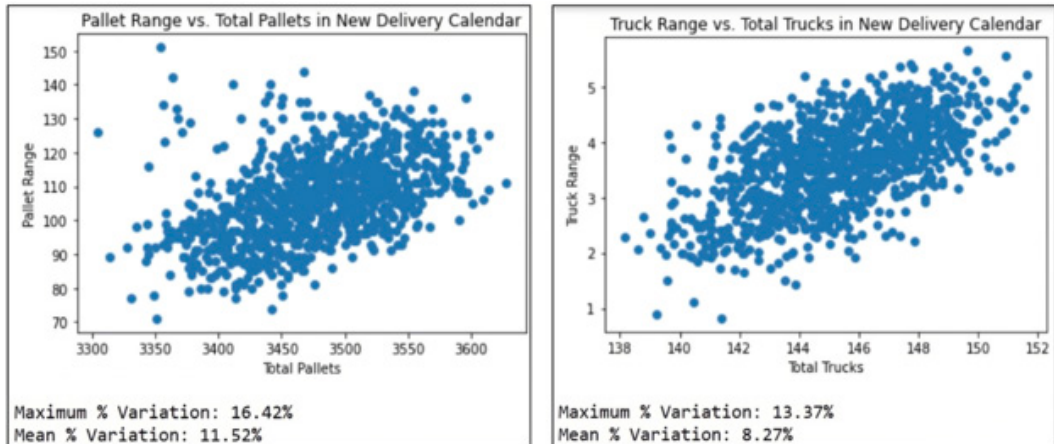
b. Following large vendor distribution rules, Tuesday, Wednesday, and Thursday (July 4th, 5th, and 6th) each get one day's supply while Friday the 7th gets 4 days' supply.
2. For each date with a delivery, check that the delivered number of trucks plus the number of trucks already relegated to that day is less than the DC capacity, meaning that delivering the vendor's trucks would not make the DC process more truckloads than it should. Finish with this vendor if this condition is satisfied for all days. Table 21 is updated to produce Table 22 in the example below.
   a. Delivering on July 4th would cause daily trucks to exceed desired limits (10.1 trucks > 10 truck limit). So, proceed to step 3.
3. For any dates not satisfying the condition in step 2, change that day's availability to *None* so it cannot be chosen as a delivery day and change delivery days back to ones. To prevent skipping vendors, this process stops if removing a day's availability would leave the vendor with no days left. Finally, re-distribute the vendor's trucks with the updated schedule. Table 23 shows the sample vendor's redistribution results, now only needing to cover six days of demand instead of seven.

Delivery Adjustments With DC Limits

Since restricted days will usually have fewer trucks than unrestricted days, restricted days should not be considered in the algorithm's adjustment process to avoid attempts to add excessive deliveries to dates that are meant to be limited. Therefore, the correction process will only attempt to level deliveries for unrestricted days (which can be determined by selecting only days in the DC Truck Capacity table marked *None*), but its logic remains the same.

Strengths, Limitations, and Impact of the New Algorithm

As evidenced by the Monte Carlo analysis, the new algorithm for MPRS generates a reasonably level load of truck deliveries with an acceptable level of variation over a range of real-world variability in order volume and transportation performance. The algorithm is in production and represents an improvement over the previous situation. The below points capture the strengths and limitations of the current model.

Table 19. Sample DC Truck Receiving Capacities

	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
Trucks	0	*None*	10	*None*	25	25	*None*

Strengths

- **Consistency.** Because the algorithm follows the same logic each time and optimizes each week separately, the results for each week will remain the same each time.
- **Accuracy.** As shown in the sensitivity analysis, the algorithm produces receiving schedules that can generally account for variability in truck deliveries without significantly impacting the leveling of DC workdays, though some manual balancing may be necessary for extreme cases.
- **Adaptability.** Since the algorithm takes several inputs, it can easily adjust and create new schedules for other situations.

Limitations

Possibly Unfeasible Deliveries. The adjustment process will usually result in a schedule that requires several vendors to deliver on days they normally do not. This could result in vendors needing to have their deliveries changed.

While the algorithm is not without limitations, it has made a tremendous impact on inbound delivery scheduling when placing POs for the quick-service restaurant chain. Since this algorithm can be easily run using existing data tables in the company's software system, the process has been automated to produce schedules on a weekly basis. Therefore, minimal additional work is necessary to reach an optimized schedule based on company preferences, which has consistently saved 8-12 hours each week for the lead planner, depending on the demand change and number of holidays or irregular days.

CONCLUSION

Inbound delivery scheduling plays a vital role in ensuring smooth and efficient DC operations. This paper introduces the concept of MPRS and presents a novel MPRS algorithm to optimize PO due dates. The algorithm helps level DC receiving calendars while factoring in constraints from holidays and workdays for DCs and vendors. By categorizing vendors by shipment volume to distribute their deliveries evenly and then adjusting to smoothly spread deliveries over the week, the algorithm creates a robust leveled delivery calendar. Through a detailed Monte Carlo simulation, the algorithm's robustness was tested, demonstrating its ability to optimize purchase order delivery dates in real-world conditions. An extension for implementing DC maximum unloading capacities into the truck distribution algorithm was also discussed.

By allowing companies to input their workdays and delivery preferences, this algorithm can be applied to any DC. Its consistency and accuracy, in the case of the quick-service restaurant chain, led to reductions of 8-12 hours each week for the company's lead planner. In addition, it produced results superior to the time-consuming manual approach. This research should help advance the practice of inbound logistics and stimulate further research on MPRS and its impact on this overlooked yet critical area of supply chain management.

Table 20. Sample DC Daily Trucks

	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
Trucks	H	11.25	9.5	15	11.5	12	14.75

Table 21. Sample Large Vendor Original Distribution

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
8	4	H	*None*	0.6	0.6	0.6	2.25	*None*

Table 22. Evaluation of Delivery Acceptability Based on Table 19 Limits

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
8	4	H	*None*	0.6	0.6	0.6	2.25	*None*

Table 23. Sample Vendor's Redistribution Results

Vendor ID	Trucks	7-2-2023Sun	7-3-2023Mon	7-4-2023Tue	7-5-2023Wed	7-6-2023Thu	7-7-2023Fri	7-8-2023Sat
8	4	H	*None*	*None*	0.67	0.67	2.67	*None*

COMPETING INTERESTS

The authors of this publication declare there are no competing interests.

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